Math 115

Summer 2017

Lecture 11

Special Products:

1) $(A+B)^2 = (A+B)(A+B)$

$= A^2 + AB + AB + B^2$

$= A^2 + 2AB + B^2$

$(x+5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2$

$= x^2 + 10x + 25$

$(2x+7)^2 = (2x)^2 + 2(2x)(7) + 7^2$

$= 4x^2 + 28x + 49$

$(3x + 8y)^2 = (3x)^2 + 2(3x)(8y) + (8y)^2$

$= 9x^2 + 48xy + 64y^2$

$(4x^2 + 3)^2 = (4x^2)^2 + 2(4x^2)(3) + (3)^2$

$= 16x^4 + 24x^2 + 9$

$(5x^3 + 2x)^2 = (5x^3)^2 + 2(5x^3)(2x) + (2x)^2$

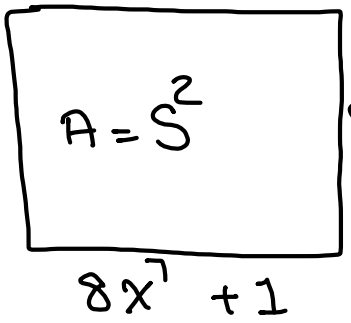
$= \boxed{25x^6 + 20x^4 + 4x^2}$

Your turn:

$(7x^4 + 10y^6)^2 = (7x^4)^2 + 2(7x^4)(10y^6) + (10y^6)^2$

$= \boxed{49x^8 + 140x^4y^6 + 100y^{12}}$

Find the Area:



$A = (8x^7 + 1)^2$

$= (8x^7)^2 + 2(8x^7)(1) + 1^2$

$\boxed{A = 64x^{14} + 16x^7 + 1}$

2) $(A-B)^2 = (A-B)(A-B)$

$= A^2 - AB - AB + B^2$

$= A^2 - 2AB + B^2$

$(x-4)^2 = (x)^2 - 2(x)(4) + (4)^2$

$= \boxed{x^2 - 8x + 16}$

$(2x-9)^2 = (2x)^2 - 2(2x)(9) + (9)^2$

$= \boxed{4x^2 - 36x + 81}$

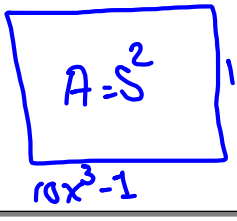
$(7x - 3y)^2 = (7x)^2 - 2(7x)(3y) + (3y)^2$

$= \boxed{49x^2 - 42xy + 9y^2}$

$(5x^2 - 2x^5)^2 = (5x^2)^2 - 2(5x^2)(2x^5) + (2x^5)^2$

$= \boxed{25x^4 - 20x^7 + 4x^{10}}$

Find area:



$A = (10x^3 - 1)^2$

$= (10x^3)^2 - 2(10x^3)\cdot 1 + 1^2$

$= 100x^6 - 20x^3 + 1$

3) $(A+B)(A-B) = A^2 - \cancel{AB} + \cancel{AB} - B^2$

Conjugates

$= A^2 - B^2$

$(x+5)(x-5) = x^2 - 5^2$

$= \boxed{x^2 - 25}$

$(2x-7)(2x+7) = (2x)^2 - (7)^2$

Conjugates

$= \boxed{4x^2 - 49}$

$(10x^6 + 3y^4)(10x^6 - 3y^4)$

$= (10x^6)^2 - (3y^4)^2 = \boxed{100x^{12} - 9y^8}$

$(12x^7 - 5x^3)(12x^7 + 5x^3)$

$= (12x^7)^2 - (5x^3)^2$

$= 144x^{14} - 25x^6$

$(5x^4 - 2x^3)(5x^4 + 2x^3)$

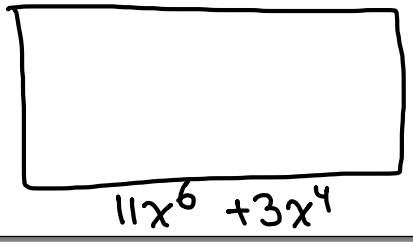
1) use FOIL: $= 25x^8 + \cancel{10x^7} - \cancel{10x^7} - 4x^6$

$= 25x^8 - 4x^6$

2) use Special products: $= (5x^4)^2 - (2x^3)^2$

$= 25x^8 - 4x^6$

Find Area



$A = (11x^6 + 3x^4)(11x^6 - 3x^4)$

$A = (11x^6)^2 - (3x^4)^2$

$= 121x^{12} - 9x^8$

Simplify

$(x - 2)(x + 2)(x^2 + 4)(x^4 + 16)$

Conjugates

$= (x^2 - 4)(x^2 + 4)(x^4 + 16)$

Conjugates

$= (x^4 - 16)(x^4 + 16)$

$= (x^4)^2 - (16)^2 = \boxed{x^8 - 256}$

Simplify

$(x^2+4x+4)(x^2-4x+4)$

$= x^4 - 4x^3 + 4x^2 + 4x^3 - 16x^2 + 16x + 4x^2 - 16x + 16$

$= \boxed{x^4 - 8x^2 + 16}$

$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

$= (x^3 - 8)(x^3 + 8)$

Conjugates

$= (x^3)^2 - (8)^2 = \boxed{x^6 - 64}$

Simplify

$$(3x-2)(2x+3)(x^2-x)$$

$$= (6x^2+5x-6)(x^2-x)$$

$$= 6x^4 - 6x^3 + 5x^3 - 5x^2 - 6x^2 + 6x$$

$$= \boxed{6x^4 - x^3 - 11x^2 + 6x}$$

Reduce

$$\frac{35x^{12}}{5x^4} = 7x^{12-4} = \boxed{7x^8}$$

$$\frac{\cancel{24}^{6}\, x^{18} y^3}{-\cancel{4}_{1}\, x^{10} y^7} = \boxed{-\frac{6x^8}{y^4}} = \boxed{\frac{-6x^8}{y^4}}$$

$$\frac{\cancel{75}^{15}\, x^{-5} y^{13}}{-\cancel{40}_{8}\, x^3 y^5} = \frac{-15\, y^{13}}{8 x^3 x^5 y^5} = \boxed{\frac{-15 y^8}{8x^8}}$$

Divide multiple terms by one term:

$$\frac{40x^7 - 10x^4}{5x^2} = \frac{40x^7}{5x^2} - \frac{10x^4}{5x^2}$$

$$= \boxed{8x^5 - 2x^2}$$

$$\frac{24x^{12} - 36x^8 + 6x^2}{6x^2}$$

$$= \frac{\cancel{24}^{4}x^{12}}{\cancel{6}x^2} - \frac{\cancel{36}^{6}x^8}{\cancel{6}x^2} + \boxed{\frac{6x^2}{6x^2}} = 4x^{10} - 6x^6 + 1$$

Your turn:

$$\frac{49x^5y^4 - 28x^3y^2}{7x^2y^2}$$

$$= \frac{\cancel{49}^{7}x^5y^4}{\cancel{7}x^2y^2} - \frac{\cancel{28}^{4}x^3\cancel{y^2}}{\cancel{7}x^2\cancel{y^2}} = \boxed{7x^3y^2 - 4x}$$

$$\frac{50x^4 - 20x^2 + 10x}{15x^2}$$

$$= \frac{50x^4}{15x^2} - \frac{20x^2}{15x^2} + \frac{10x}{15x^2} = \boxed{\frac{10x^2}{3} - \frac{4}{3} + \frac{2}{3x}}$$

Dividing multiple terms by 2 or more terms:

$$\frac{x^2 - 7x + 10}{x - 2}$$

Long Division

$x \boxed{x} = x^2$

$x \boxed{-5} = -5x$

$$x - 5$$

$$x - 2 \overline{)\, x^2 - 7x + 10}$$

x

$-(x^2 - 2x\ \ )$

$-5x + 10$

$-(-5x + 10)$

Remainder $\rightarrow 0$

Divide $\dfrac{x^3 - 2x + 4x^2 - 3}{x + 1}$

$$x + 1 \overline{)\, x^3 + 4x^2 - 2x - 3}$$

x^2

$-(x^3 + x^2\ \ \ \ \ \ \ \ \ \ \ \ )$

$3x^2 - 2x - 3$

$-(3x^2 + 3x\ \ \ \ \ \ \ \ \ \ \ \ )$

$-5x - 3$

$-(-5x - 5)$

Rem. $\rightarrow 2$

$x \boxed{x^2} = x^3$

$x \boxed{3x} = 3x^2$

$x \boxed{-5} = -5x$

$$x^2 + 3x - 5 + \frac{2}{x+1}$$

Divide $\dfrac{2x^2 - 6x + 4}{2x - 1}$

$2x \boxed{x} = 2x^2$

$2x \boxed{-5/2} = -5x$

$4 - \frac{5}{2} = \frac{8}{2} - \frac{5}{2} = \frac{3}{2}$

$$x - \frac{5}{2} + \frac{\frac{3}{2}}{2x-1}$$

$$2x-1 \overline{)\, 2x^2 - 6x + 4}$$

$x - 5/2$

$-(2x^2 - x)$

$-5x + 4$

$-(-5x + 5/2)$

Rem. $\rightarrow \frac{3}{2}$

Your turn

$$\frac{x^3 - 7x^2 + 5x + 1}{x + 3}$$

$x \boxed{x^2} = x^3$

$x \boxed{-10x} = -10x^2$

$x \boxed{35} = 35x$

$$x+3 \overline{)\, x^3 - 7x^2 + 5x + 1}$$

$x^2 - 10x + 35$

$-(x^3 + 3x^2)$

$-10x^2 + 5x + 1$

$-(-10x^2 - 30x)$

$35x + 1$

$-(35x + 105)$

-104

Always

$$x^2 - 10x + 35 + \frac{-104}{x+3}$$

Divide $\frac{x^2 \quad -16}{x+4} = \frac{x^2 + 0x \quad -16}{x+4}$

$$\begin{array}{r} x \quad -4 \\ x+4 \overline{\big)\, x^2 + 0x \quad -16} \\ -(x^2 \quad +4x \qquad) \\ \hline -4x \quad -16 \\ -(-4x \quad -16) \\ \hline 0 \end{array}$$

$x\,\boxed{x} = x^2$

$x\,\boxed{-4} = -4x$

$x - 4$

$\frac{x^3 \quad +5x \quad -3}{x-2}$

$$\begin{array}{r} x^2 \quad +2x \quad +9 \\ x-2 \overline{\big)\, x^3 + 0x^2 + 5x \; -3} \\ -(x^3 \quad -2x^2 \qquad) \\ \hline 2x^2 \quad +5x \; -3 \\ -(2x^2 \quad -4x \quad) \\ \hline 9x \; -3 \\ -(9x \; -18) \\ \hline 15 \end{array}$$

$x\,\boxed{x^2} = x^3$

$x\,\boxed{2x} = 2x^2$

$x\,\boxed{9} = 9x$

$x^2 + 2x + 9 + \frac{15}{x-2}$

Divide: $\dfrac{-9x^5+3x^4-12}{3x^3}$

$= \dfrac{-9x^5}{3x^3} + \dfrac{3x^4}{3x^3} - \dfrac{12}{3x^3} = \boxed{-3x^2+x-\dfrac{4}{x^3}}$

Divide: $\dfrac{4x^2-13x-12}{4x+3}$

$4x \boxed{x} = 4x^2$

$4x \boxed{-4} = -16x$

$x-4$

$$\begin{array}{r} x-4 \\ 4x+3 \overline{)\,4x^2-13x-12} \\ -(4x^2+3x) \\ \hline -16x-12 \\ -(-16x-12) \\ \hline 0 \end{array}$$

Divide: $\dfrac{8y^6-3y^2-4y}{-4y}$

$= \dfrac{8y^6}{-4y} - \dfrac{3y^2}{-4y} - \dfrac{4y}{-4y} = \boxed{-2y^5+\dfrac{3y}{4}+1}$

Divide: $\dfrac{7x^2-3}{x-3}$

$$\begin{array}{r} 7x+21 \\ x-3 \overline{)\,7x^2+0x-3} \\ -(7x^2-21x) \\ \hline 21x-3 \\ -(21x-63) \\ \hline 60 \end{array}$$

$7x+21+\dfrac{60}{x-3}$

Class QZ

1) Simplify: $(-2x^4)^3$

2) Multiply & Simplify: $(2x-3)(2x+3)+9$

3) Simplify: $\frac{-36x^8y^3}{12x^2y^{10}}$

Due Monday: SG 12, 13, 14

Agenda for Monday:

1) Lecture (3 hrs)

2) Exam II

Monomial:

Number times Variables that can have exponent of non-negative integers.

$5x^3$ Coef. $= 5$, Deg. $= 3$

$-4x^2y^4$ Coef. $= -4$, Deg. $= 2+4 = 6$

$\frac{2}{3}x^6y^3z$ Coef. $= \frac{2}{3}$, Deg. $= 6+3+1 = 10$

$-100xyzt$ Coef. $= -100$; Deg. $= 1+1+1+1 = 4$

2017 → No variable
→ Constant
→ Deg. $= 0$

Binomial : + or − of two monomials

$4x + 10$ D = 1, L.C. = 4, Const. = 10

$-3x^2 - 7x$ D = 2, L.C. = −3, No Constant

$\frac{2}{3}x^4 + \frac{3}{5}y^{10}$ D = 10, L.C. $\frac{3}{5}$, //

$6x^3 - 3x^6$ D = 6, L.C. = −3

Monomials	D	C
$5x^8$	8	5
$-20x^2y^3$	2+3 = 5	-20
1337	0 Constant.	

$7x^3y^7 - 12x^2y$

Monomial	D	C
$7x^3y^7$	10	7
$-12x^2y$	3	-12

D = 10
L.C. 7

Trinomials: +, − of 3 monomials

$x^2 + 8x - 12$ D=2
L.C. = 1
Const. −12

$-6x^4y^6 + 100x^3y^2 - 400xy$

$-6x^4y^6$: D = 10, C = −6
$100x^3y^2$: D = 5, C = 100
$-400xy$: D = 2, C = −400

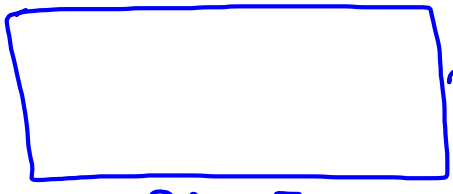
Deg. 10
L.C. −6

$25x^8y^7$ $-35x^4y^3$ $+2017$

D = 15, C = 25 | D = 7, C = -35 | Constant, D = 0

Find Area & Perimeter

$A = LW$, $P = 2L + 2W$



$A = (2x+7)(x-2)$

$= 2x^2 + 3x - 14$

Trinomial, D = 2, L.C. = 2, Const. = -14

$P = 2(2x+7) + 2(x-2)$

$= 6x + 10$

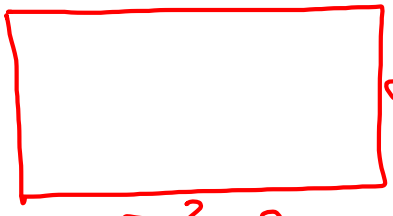
Binomial, D = 1, L.C. = 6, Const = 10

Find A & P

$A = LW$

$= (5x^2+3)(5x^2-3)$

$= 25x^4 - 9$



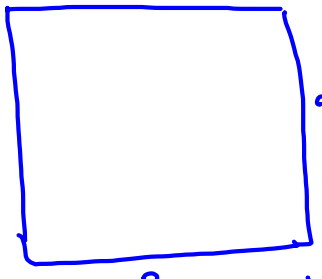
$P = 2L + 2W$

$P = 2(5x^2+3) + 2(5x^2-3)$

$= 20x^2$

Binomial, D = 4, L.C. = 25, Const = -9

Monomial, D = 2, C = 20



Find A

$A = S^2$

$= (x^2-3x+4)^2$

$= (x^2-3x+4)(x^2-3x+4)$

$= x^2(x^2-3x+4) \; -3x(x^2-3x+4) \; +4(x^2-3x+4)$

$= x^4 \; -3x^3 \; +4x^2 \; -3x^3 \; +9x^2 \; -12x \; +4x^2 \; -12x \; +16$

$= x^4 - 6x^3 + 17x^2 - 24x + 16$

Polynomial

D = 4

L.C. = 1

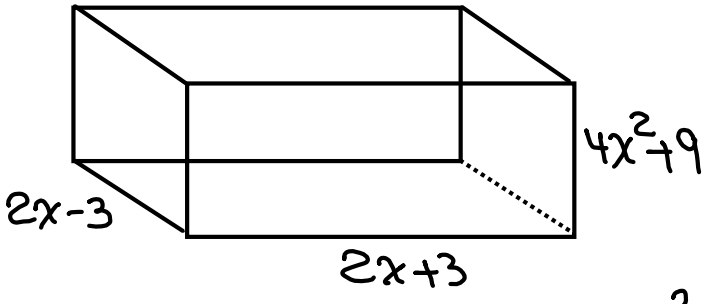
Const = 16

Polynomial with 1 term ⇒ Monomial

2 terms ⇒ Binomial

3 terms ⇒ Trinomial

Find the Volume:



$V = LWH$

Binomial

D = 4, LC = 16

Const. -81

$V = (2x-3)(2x+3)(4x^2+9) = 16x^4 - 81$

$(2x-3)(2x+3)(4x^2+9)$

Conjugates

$=[(2x)^2-(3)^2](4x^2+9)$

$=(4x^2-9)(4x^2+9)$

conjugates

$=(4x^2)^2-(9)^2$

$=16x^4-81$

Find eqn of a line that contains $(3,-4)$ and is parallel to the line $2x-3y=-9$.

$\Rightarrow -3y=-2x-9$

$y=\frac{2}{3}x+3$

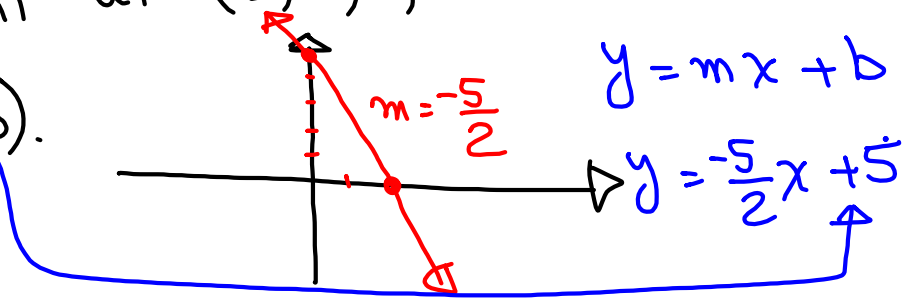
$y-y_1=m(x-x_1)$

$y-(-4)=\frac{2}{3}(x-3)$

$y+4=\frac{2}{3}x-2$

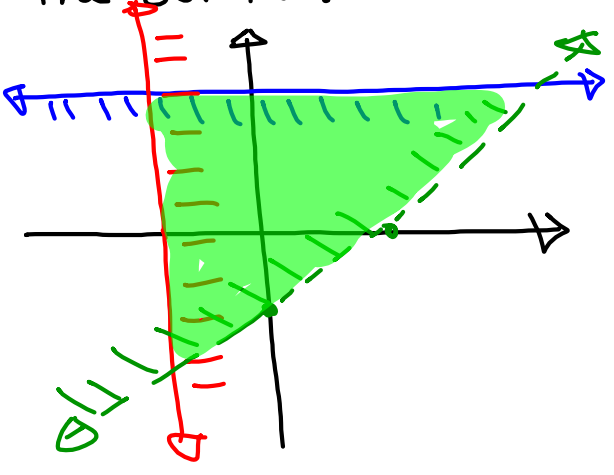
$\Rightarrow \boxed{y=\frac{2}{3}x-6}$

Find eqn of a line that has x-Int at $(2,0)$, and y-Int at $(0,5)$.



Graph & Shade the Solution

$$\begin{cases} y \le 4 \\ x \ge -3 \\ y > \frac{3}{4}x - 3 \end{cases}$$



$$\begin{cases} 4 \cdot (2x - 3y = 7) \\ 3 \cdot (5x + 4y = 12) \end{cases} \Rightarrow 23x = 64$$

$$x = \frac{64}{23}$$

Solve for x.

Clear Your next weekend.